FREE JEE MAIN 2014

SOLUTIONS

STANDARD ANSWER KEY											
Q	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
A	B	B	B	B	B	A	A	B	D	A	C
Q	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
A	A	D	D	C	D	C	C	B	B	D	C
Q	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**
A	A	B	C	C	A	B	D	B	B	B	A
Q	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**
A	C	A	A	A	C	B	D	B	A	B	A
Q	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**
A	A	D	C	C	D	C	A	B	B	C	D
Q	**56**	**57**	**58**	**59**	**60**	**61**	**62**	**63**	**64**	**65**	**66**
A	C	A	B	A	D	D	B	B	B	B	C
Q	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**
A	D	C	B	B	C	A	A	B	A	D	A
Q	**78**	**79**	**80**	**81**	**82**	**83**	**84**	**85**	**86**	**87**	**88**
A	A	B	B	D	B	B	C	A	C	C	C
Q	**89**	**90**									
A	A	A									

PART A – CHEMISTRY

(1) **(B)** $\wedge^{\infty}_{H_2O} = \dfrac{k \times 1000}{N}$, $N = \dfrac{5.5 \times 10^{-7} \times 1000}{550} = 10^{-6}$

$K_w = N^2$ or M^2, $pH = pOH = 6$

(2) **(B)** Let V ml of RNH_3Cl added into RNH_2 solution

$[RNH_3Cl]$ in resultant solution $= \dfrac{0.2 \times V}{100 + V}$

$$[RNH_2] = \frac{100 \times 0.1}{100 + V}; \quad pOH = pK_b + \log\frac{[RNH_3^+]}{[RNH_2]}$$

$$5.3 = 5 + \log\left[\frac{\left(\dfrac{0.2 \times V}{100 + V}\right)}{\dfrac{100 \times 0.1}{100 + V}}\right]$$

$$\therefore\ 2 = \frac{0.2 \times V}{10} \Rightarrow V = 100\,\text{ml.}$$

(3) **(B)**

(4) **(B)**

$$N_2O_5(g) \rightleftharpoons N_2O_3(g) + O_2(g)$$

con^n at eq^m $\quad 2 - x \quad x - y \quad x + y$

$$N_2O_3(g) \rightleftharpoons N_2O(g) + O_2(g)$$

con^n at eq^m $\quad x - y \quad y \quad y + x$

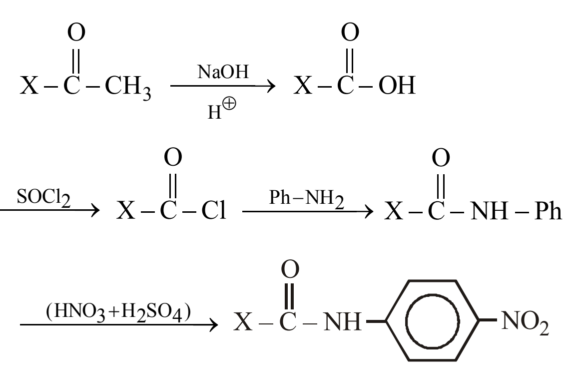
$$2.5 = \frac{2.5 \times (x - y)}{(2 - x)}$$

$x - y = 2 - x$ or $2x - y = 2$

& as per given $[O_2(g)] = x + y = 2.5$

$x = 1.5$ and $[N_2O(g)] = y = 1.0$

(5) **(B)**



(6) **(A)** Equivalent of $MoO_3 = \dfrac{M}{6} = \dfrac{144}{6} = 24$

equivalent of H_2: $\dfrac{192}{24} = \dfrac{W_{H_2}}{1} \Rightarrow W_{H_2} = 8gm.$

(7) **(A)** $\frac{1}{2}N_2(g)+\frac{1}{2}O_2(g)\longrightarrow NO(g)$; $\Delta_f H°=90.2$

$N_2(g)+O_2(g)\longrightarrow 2NO(g)$; $\Delta_f H°=90.2\times 2$ ...(1)

$2NO(g)+O_2(g)\longrightarrow 2NO_2(g)$; $\Delta_f H°=-114$ ...(2)

$2NO_2(g)+\frac{1}{2}O_2(g)\longrightarrow N_2O_5(g)$;

$\Delta_f H°=\frac{-102.6}{2}=-51.3$ ...(3)

Eq. (1) + (2) + (3)

$N_2(g)+\frac{5}{2}O_2(g)\longrightarrow N_2O_5(g)$

$\Delta_f H°(N_2O_5, g)=15.1$ kJ/mol

(8) **(B)** $4S^3=3.2\times 10^{-11}$

$S=2\times 10^{-4}$

Let solubility of CaF_2 is x in KF solution.

$K_{sp}=x(2x+4\times 10^{-3})^2$; $3.2\times 10^{-11}=x\times 10^{-6}\times 16$

$x=2\times 10^{-6}$

(9) **(D)** (A) Tranquillizers are the substances used for the treatment of mental diseases. These act on higher centres of the central nervous system. These are also called psychotherapeutic drugs.

(B) Vitamin C, Vitamin E and β-carotene are antioxidants (substances that act against oxidants).

(C) Sodium or potassium salt of a long chain fatty acid is called soap.

(10) **(A)**

	$KMnO_4$	MnO_2	Mn^{2+}	$Mn(OH)_3$	MnO_4^{2-}
O.S of Mn	+7	+4	+2	+3	+6
electrons needed	0	3	5	4	1

(11) **(C)** $w=Z\times n\times q$

$Z=\frac{96.5}{0.9\times 2\times 96500}=5.55\times 10^{-4}$

(12) **(A)** $[H^+]_{remaining}=\frac{5\times 0.08-10\times 0.03}{10+5+485}=2\times 10^{-4}$

pH = 3.7

(13) **(D)** Ester of dicarboxylic acids undergo an intramolecular version of the Classian condensation when a five or six membered ring can be formed. This reaction is an example of a Dieckmann's condensation.

(14) **(D)** (A) The change of atmospheric temperature with altitude is called the lapse rate.

(B) The gases responsible for greenhouse effects are CO_2, Water vapour, CH_4 and ozone.

(C) The order of contribution to the acid rain is

$H_2SO_4 > HNO_3 > HCl$

(15) **(C)** Cell reaction is

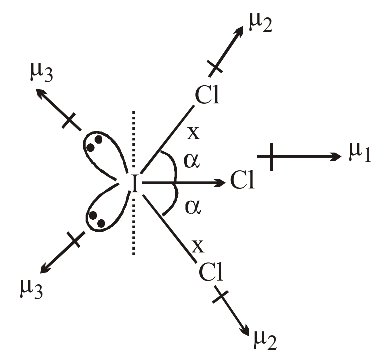
$2Ag(s)+2H^+(aq)\rightarrow 2Ag^+(aq)+H_2(g)$

$E_{cell}=-0.80-\frac{0.0591}{2}\log\frac{(0.1)^2\times 0.1}{(0.1)^2}$

$E_{cell}=-0.80+0.02955=-0.77$ V

(16) **(D)** $k=\frac{x}{t}$; $t=x.\frac{1}{k}$; $t_{3/4}=\frac{3}{4k}.a$

(17) **(C)**



$(\mu_D \neq 0)$ x > y

Molecule is polar and planar.

Both ∠ Cl I Cl are equal

Equatorial I – Cl bond has more s-character than axial I – Cl bond.

(18) **(C)** Oxidation of secondary alcohol.

(19) **(B)** Fe and Pt do not form amalgam

$\underset{\text{Silver amalgam}}{Ag-Hg}\xrightarrow[\Delta]{\text{Fe vessel}}Ag+Hg\uparrow$

Vessel made of other metal will form amalgam with liberated mercury.

(20) **(B)** (A) $NH_4NO_2\xrightarrow{\Delta}N_2+2H_2O$

(B) $(NH_4)_2SO_4\xrightarrow{\Delta}NH_3+H_2SO_4$

(C) $2NH_4ClO_4\xrightarrow{\Delta}N_2+Cl_2+2O_2+4H_2O$

(D) $(NH_4)_2Cr_2O_7\xrightarrow{\Delta}N_2+Cr_2O_3+4H_2O$

(21) **(D)** (A) The size of the colloidal particles is in between the size of the molecule and the size of the particle of a coarse suspension.

(B) White portion of the egg is condensed colloidal solution whereas the sol prepared from this substance is a dilute colloidal solution.

(C) Dialysis is a slow process. It may some times take several days for its completion.

(22) **(C)** $P=P_A^0x_A+P_B^0x_B$

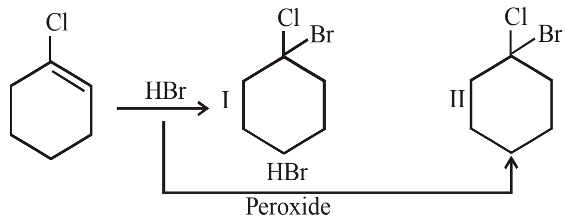
$600=P_A^0\left(\frac{3}{3+2}\right)+P_B^0\left(\frac{2}{2+3}\right)$; $3P_A^0+2P_B^0=3000$

$630=P_A^0\left(\frac{4.5}{4.5+2+0.5}\right)+P_B^0\left(\frac{2}{4.5+2+0.5}\right)$

$4.5\,P_A^0+2\,P_B^0=4410$

$1.5P_A^0=1410$; $P_A^0=940$ and $P_B^0=90$

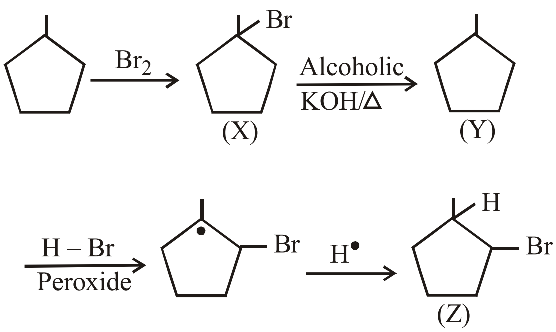
(23) (A)



(24) (B) $HC \equiv CH \xrightarrow[H_2SO_4]{HgSO_4} CH_3-CHO \xrightarrow{aldol}$

$$\underset{3}{CH_3}-\overset{OH}{\underset{2}{CH}}-\underset{1}{CH_2}-CHO \text{ (3-hydroxy butanal)}$$

(25) (C)



(26) (C) (A) When a dilute solution of an acid is added to a dilute solution of a base, neutralization reaction takes place.

(B) In acid-base titrations, at the end point, the amount of acid becomes chemically equilvalent to the amount of base present.

(C) In the case of a strong acid and a strong base titration, at the end point the solution becomes neutral (i.e. pH = 7)

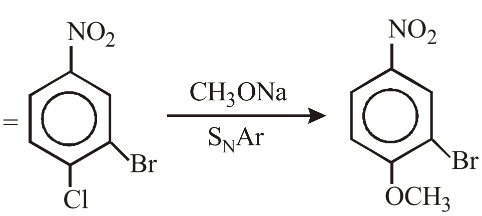
(D) In acid-base titrations the end point is determined by the hydrogen ion concentration of the solution.

(27) (A) Since $K.E. = \frac{1}{2}mv^2$ and $\lambda = \frac{h}{mv}$.

$\therefore K.E. = \frac{1}{2}m.\frac{h^2}{m^2\lambda^2} = \frac{h^2}{2m\lambda^2}$. As λ is the same.

$\therefore K.E. \propto \frac{1}{m}$

(28) (B) X =



(29) (D) Nitric acid decomposes to give NO_2 which is brown.

$$4HNO_3 \rightarrow 4NO_2 + 2H_2O + O_2$$

When air is passed through acid the reaction proceeds towards left hand hand side and brown colour diminishes.

(30) (B) The octet around N is complete, hence it has no electrophilic character. N has no unshared pair of electrons to act as an nucleophile

PART B – PHYSICS

(31) (B) Here we have $\frac{Qq}{a} + \frac{q^2}{a} + \frac{Qq}{a\sqrt{2}} = 0$

or $Q\sqrt{2} + q\sqrt{2} + Q = 0$

or $Q(\sqrt{2}+1) = -q\sqrt{2}$

$\therefore Q = -\frac{q\sqrt{2}}{\sqrt{2}+1} = -\frac{2q}{2+\sqrt{2}}$

(32) (B) $\Delta Q = \frac{f}{2}nRT + nRT = \frac{3}{2}nRT + nRT = \frac{5}{2}nRT$

$\therefore$ Fraction $= \frac{(3/2)\,nRT}{(5/2)\,nRT} = \frac{3}{5}$

(33) (A) Least count of vernier callipers

= value of one division of main scale – value of one division of vernier scale

Now, $N \times a = (N+1)\,a'$

(a' = value of one division of vernier scale)

$a' = \frac{N}{N+1}a$

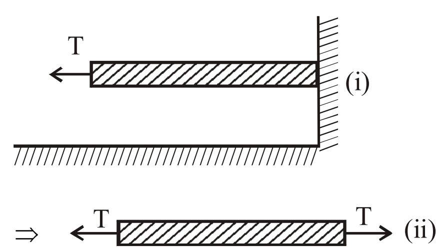
$\therefore$ Least count $= a - a' = \frac{a}{N+1}$

(34) (C) $E_1 = \frac{1}{2}kx^2, E_2 = \frac{1}{2}ky^2,$

$E = \frac{1}{2}k(x+y)^2 = E_1 + E_2 + 2\sqrt{E_1E_2}$

$= 2 + 8 + 2\sqrt{16} = 18J$

(35) (A) Tension in both strings shall be same which can be observed by making FBD of string in figure (i).



FBD of string in figure (i)

(36) **(A)** As long as the block of mass m remains stationary, the block of mass M released from rest comes down by $\frac{2Mg}{K}$ (before coming it rest momentarily again).
Thus the maximum extension in the spring is

$$x = \frac{2Mg}{k} \quad \text{..........(1)}$$

For block of mass m to just move up the incline
$kx = mg \sin\theta + \mu\, mg \cos\theta$ ....(2)

$$2Mg = mg \times \frac{3}{5} + \frac{3}{4} mg \times \frac{4}{5} \quad \text{or} \quad M = \frac{3}{5}m$$

(37) **(A)** For a particle undergoing SHM with an amplitude A and angular frequency ω, the maximum acceleration $= \omega^2 A$
Here the maximum force on the particle $= QE_0$

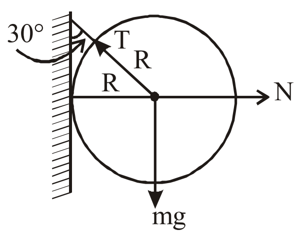
$$\therefore \text{ maximum acceleration} = \frac{QE_0}{m} = \omega^2 A$$

$$\therefore\ A = \frac{QE_0}{m\omega^2}$$

(38) **(C)** $mg = T\cos 30°$

$N = T \sin 30°$

$$\Rightarrow N = \frac{mg}{\sqrt{3}}$$



(39) **(B)** S_1 : No work is done by net force, it only changes direction of momentum of particle. Hence S_1 is false.
S_2 : True by definition.
S_3 : Nothing is said about acceleration of both particles. Hence angle between velocity and acceleration of centre of mass may not be zero. Consequently centre of mass may not move along a straight line. Hence S_3 is false.

$$S_4 : \vec{V}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + ... + m_n\vec{v}_n}{m_1 + m_2 + ... + m_n} = \frac{\vec{F}_{net}}{(m_1 + m_2 + ... + m_n)}$$

Direction at $\vec{P}_{net}$ is fixed so $\vec{V}_{cm}$ is also constant in the direction. So path of CM will be straight line.

(40) **(D)** $\frac{1}{2}mv^2 = \frac{hc}{\lambda} - \phi$

$$\frac{1}{2}mv'^2 = \frac{hc}{(3\lambda/4)} - \phi = \frac{4hc}{3\lambda} - \phi. \text{ Clearly, } v' > \sqrt{\frac{4}{3}}v$$

(41) **(B)** Given : $\ell = 1\text{m}, B = 5 \times 10^{-3}\ \text{Wb/m}^2$

$$f = \frac{1800}{60} = 30 \text{ rotations/sec}$$

In one rotation, the moving rod of the metal traces a circle of radius $r = \ell$

$\therefore$ Area swept in one rotation $= \pi r^2$

$$\frac{d\phi}{dt} = \frac{d}{dt}(BA) = B.\frac{dA}{dt} = \frac{B\pi r^2}{T}$$

$= B f \pi r^2 = (5 \times 10^{-3}) \times 3.14 \times 30 \times 1 = 0.471\ \text{V}$
$\therefore$ e.m.f. induced in a metal rod $= 0.471$ V

(42) **(A)** Charge conservation is violated in [B, C, D], nucleon conservation is violated in (D), (A) works.

(43) **(B)** Using $\frac{A}{A_0} = \left(\frac{1}{2}\right)^{t/T}$ or $\frac{1}{32} = \left(\frac{1}{2}\right)^{7.5/T}$

or $\left(\frac{1}{2}\right)^5 = \left(\frac{1}{2}\right)^{7.5/T}$ or $5 = \frac{7.5}{T}$ i.e. T = 1.5 hours

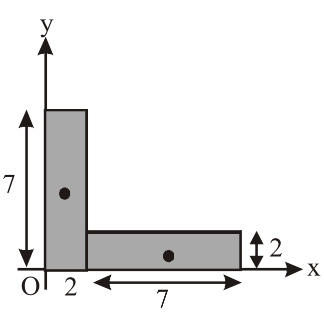
(44) **(A)** Divide the ring into infinitely small lengths of mass dm_1. Even though mass distribution is non-uniform, each mass dm_1 is at same distance R from origin.
$\therefore$ MI of ring about z-axis is
$= dm_1R^2 + dm_2 R^2 + ...+ dm_nR^2 = MR^2$

(45) **(A)** $r = \frac{mv}{qB} \Rightarrow r \propto m$ (v = same for both charges)

(46) **(D)** $\frac{\text{Stress}}{\text{Strain}} = Y =$ slope of graph

$$\therefore\ \frac{Y_A}{Y_B} = \frac{\tan 60°}{\tan 30°} = 3 \quad \therefore\ Y_A = 3Y_B$$

(47) **(C)**



$$X_{cm} = \frac{m \times 5.5 + 1 \times m}{2m} = \frac{6.5}{2} = \frac{13}{4}\text{cm.}$$

$$Y_{cm} = \frac{m \times 3.5 + 1 \times m}{2m} = \frac{4.5}{2} = \frac{9}{4}\text{cm.}$$

(48) **(C)** $\phi = \frac{Q_{net}}{\epsilon_0}$ For dipole $Q_{net} = 0 \Rightarrow \phi = 0$

(49) **(D)** $B = \frac{\mu NiR^2}{2(R^2 + x^2)^{3/2}}$

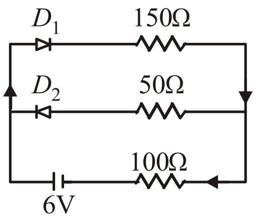
$B_{max} \Rightarrow x_{min} = 0$

(50) **(C)** $\text{Process}_{1\to2}$ and $\text{Process}_{3\to4}$ are isochoric processes.
$W_{12} = 0, W_{34} = 0, W_{23} = nR(T_3 - T_2) = 3R(1600 - 400)$
$= 3600$ R
$W_{41} = nR(T_1 - T_4) = 3R(200 - 800) = -1800$ R
$W = (3600 - 1800)R = 1800R = 15$ kJ.

(51) **(A)** $\Delta p = mv = m\sqrt{2gh}$

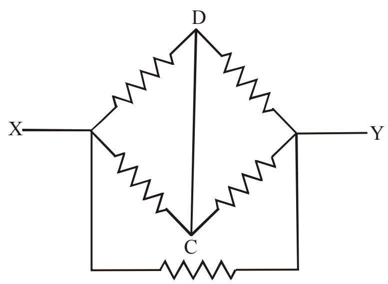
(52) **(B)** In the circuit, diode D_1 is forward biased, while D_2 is reverse biased. Therefore, current i (through D_1 and 100Ω resistance) will be

$$i = \frac{6}{50+100+150} = 0.02A$$



Here, 50Ω is the resistance of D_1 in forward biasing.

(53) **(B)**



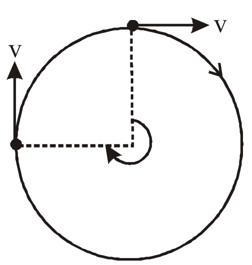
Equivalent resistance = 5Ω

(54) **(C)** At the highest position the speed of the ball is zero. Hence the only force acting on the ball of mass m is mg.

$\therefore$ Acceleration of ball at highest position is g.

(55) **(D)** The angular displacement of the particle in t = 1 sec is

$$\theta = \omega t = \frac{v}{R}t = \frac{3\pi}{2}$$



$\therefore$ The magnitude of impulse by centripetal force in t = 1 second = change in momentum

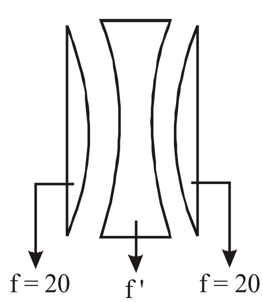
$= \sqrt{2}mv = 3\sqrt{2}\pi$ Ns

(56) **(C)** $\frac{1}{f} = (\mu-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

R = 10 cm.

$$f' = (3-1)\left(\frac{1}{-10} - \frac{1}{10}\right)$$

$$= -\frac{10}{4}$$



$$\frac{1}{f_{eq}} = \frac{1}{20} - \frac{4}{10} + \frac{1}{20} = \frac{2}{20} - \frac{4}{10}$$

$$\Rightarrow f_{eq} = -\frac{10}{3}\text{ cm}$$

(57) **(A)** $U_g = -\frac{GM_1M_2}{R}$

$$\Rightarrow U_f - U_i = -\frac{GMm}{2R} - \left(-\frac{GMm}{R}\right)$$

$$\Delta U = \frac{GMm}{2R} = \frac{mgR}{2} \quad \left\{g = \frac{GM}{R^2}\right\}$$

(58) **(B)** $T \uparrow \lambda \uparrow$; red $\rightarrow$ yellow, $\lambda_r > \lambda_y$

(59) **(A)** Initial condition

Volume = 76cm × A

Pressure = (76 + 76) dg = 152dg

Final condition

Volume = 152 cm × A

Pressure = 76dg

$\therefore$ $P_1V_1 = P_2V_2$

$\therefore$ Both points lie on the same isothermal line

i.e. both have same T.

(60) **(A)** Filling liquid will increase the optical path length by same amount.

$\therefore$ CBF will not shift.

PART C – MATHEMATICS

(61) **(D)** It passes through a fixed point (3, 4)

Slope of line joining (3, 4) and (1, –2) is – 6/–2 = 3

$\therefore$ Slope of required line = –1/3

Equation is $y - 4 = -\frac{1}{3}(x-3)$

$x + 3y - 15 = 0$

(62) **(B)** $f'(3^+) = \lim\limits_{h\to 0} \frac{f(3+h) - f(3)}{h}$

$$= \lim_{h\to 0} \frac{(2-e^h) - 1}{h} = -\lim_{h\to 0}\left(\frac{e^h - 1}{h}\right) = -1$$

$$f'(3^-) = \lim_{h\to 0} \frac{f(3-h) - f(3)}{-h}$$

$$= \lim_{h\to 0} \frac{\sqrt{10-(3-h)^2} - 1}{-h} = -\lim_{h\to 0} \frac{\sqrt{1+(6h-h^2)} - 1}{-h}$$

$$= \lim_{h\to 0} \frac{6h - h^2}{-h(\sqrt{1+6h-h^2}+1)}$$

$$= \lim_{h\to 0} \frac{h\,(h-6)}{h(\sqrt{1+6h-h^2}+1)} = \frac{-6}{2} = -3$$

Hence, $f'(3^+) \neq f(3^-)$

(63) **(B)** $2(x+iy)=\sqrt{x^2+y^2}+2i$

$2x=\sqrt{x^2+y^2}$ and $2y=2$ i.e. $y=1$

$4x^2=x^2+1$ i.e., $3x^2=1$ i.e., $x=\pm\dfrac{1}{\sqrt{3}}$

$x=\dfrac{1}{\sqrt{3}}$ $(\because x\ge 0)$ $\therefore$ $z=\dfrac{1}{\sqrt{3}}+i=\dfrac{\sqrt{3}}{3}+i$

(64) **(B)** $A^{-1}=\dfrac{1}{\det A}\text{adj } A$

$$=\frac{1}{abc}\begin{bmatrix} bc & 0 & 0\\ 0 & ca & 0\\ 0 & 0 & ab\end{bmatrix}=\begin{bmatrix}\frac{1}{a} & 0 & 0\\ 0 & \frac{1}{b} & 0\\ 0 & 0 & \frac{1}{c}\end{bmatrix}$$

The inverse of a diagonal matrix is a diagonal matrix. Both true but statement-2 is not correct explanation of statement-1.

(65) **(B)** Let A be the event such that sum is ₹ 20 or more

$\therefore$ P (A) = 1 – P (Total value is < 20)

$$=1-\frac{{}^6C_2-{}^2C_2}{{}^8C_2}=1-\frac{14}{28}=1-\frac{1}{2}=\frac{1}{2}$$

(66) **(C)** $\left(x-\dfrac{1}{5}\right)^2+\left(y-\dfrac{2}{5}\right)^2=(\lambda^2-4\lambda+4)\left(\dfrac{3x+4y-1}{5}\right)^2$

i.e., $\sqrt{\left(x-\dfrac{1}{5}\right)^2+\left(y-\dfrac{2}{5}\right)^2}=|\lambda-2|\left|\dfrac{3x+4y-1}{\sqrt{5}}\right|$

is an ellipse, if $0<|\lambda-2|<1$

i.e., $\lambda\in(1,2)\cup(2,3)$

(67) **(D)** $\theta_1+\theta_2=\dfrac{\pi}{2}$

$$\therefore\ I=\int_{\theta_1}^{\theta_2}\frac{d\theta}{1+\tan\left(\frac{\pi}{2}-\theta\right)}=\int_{\theta_1}^{\theta_2}\frac{\tan\theta\, d\theta}{1+\tan\theta}$$

and also $I=\displaystyle\int_{\theta 1}^{\theta 2}\frac{d\theta}{1+\tan\theta}$

$$\therefore\ 2I=\int_{\theta_1}^{\theta_2}d\theta=\theta_2-\theta_1=\frac{1002\pi}{2008}\Rightarrow I=\frac{501\pi}{2008}$$

Hence, $K=2008$.

(68) **(C)** $y^2=4a\left(\dfrac{y+am}{m}\right)$ i.e., $my^2-4ay-4a^2m=0$

$m\ne 0$; $16a^2+16a^2m^2>0$ which is true $\forall$ m.

$\therefore$ $m\in R-\{0\}$

(69) **(B)** For non-trivial solution,

$$\Delta=\begin{vmatrix}\sqrt{2}a & \sin B & \cos B\\ 1 & \cos B & \sin B\\ -1 & \sin B & -\cos B\end{vmatrix}=0$$

$\Rightarrow$ $a\sqrt{2}\,[-\cos^2 B-\sin^2 B]-\sin B\,[-\cos B+\sin B]$
$+\cos B\,[\sin B+\cos B]=0$

$\Rightarrow$ $-a\sqrt{2}+\sin 2B+\cos 2B=0\Rightarrow a\in[-1,1]$

(70) **(B)** S_1 : if $x^2+ax+7=0$ has imaginary roots with positive real parts then D < 0 and sum of roots > 0

$\Rightarrow a^2-28<0$ and $-a>0$

$\Rightarrow -\sqrt{28}<a<\sqrt{28}$ and $a<0$

$\Rightarrow a=-1,-2,-3,-4,-5$

S_2 : $x^2-(a+3)x+5=0$ has roots α, a, β

If α, a, β are in AP. then

$2a=\alpha+\beta\Rightarrow 2a=a+3\Rightarrow a=3$

The equation becomes $x^2-6x+5=0$ which has roots 1 and 5.

S_3 : Case- I :

If $0<x<1$, then $2+x\ge 6-x>0\Rightarrow 2x\ge 4$ and $x<6$

$\Rightarrow x\ge 2$ and $x<6\Rightarrow x\in[2,6)$

$\therefore x\in(0,1)\cap[2,6)=\phi$, $\therefore x\in\phi$

Case II : If $x>1$, then $0<2+x\le 6-x$

$\Rightarrow x>-2$ and $x\le 2$, $\therefore x\in(1,2]$

(71) **(C)** Vector $\vec{V}$ along the line of intersection of the given planes is

$$\vec{V}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & 1\\ 1 & 1 & 2\end{vmatrix}=\hat{i}(2-1)-\hat{j}(4-1)+\hat{k}(2-1)$$

$$=\hat{i}-3\hat{j}+\hat{k}$$

$\vec{u}=\hat{i}$, $\therefore$ $\cos\theta=\dfrac{1}{\sqrt{11}}$; $\theta=\tan^{-1}\sqrt{10}$

(72) **(A)** $\dfrac{{}^nC_k}{{}^nC_{k+1}}=\dfrac{1}{2}\Rightarrow\dfrac{n!}{k!\,(n-k)!}\dfrac{(k+1)!(n-k-1)!}{n!}=\dfrac{1}{2}$

or $\dfrac{k+1}{n-k}=\dfrac{1}{2}$

$2k+2=n-k$

$n-3k=2$ ....(1)

Similarly, $\dfrac{{}^nC_{k+1}}{{}^nC_{k+2}}=\dfrac{2}{3}$

$$\frac{n!}{(k+1)!\,(n-k-1)!}.\frac{(k+2)!(n-k-2)!}{n!}=\frac{2}{3}$$

$$\frac{k+2}{n-k-1}=\frac{2}{3}$$

$3k + 6 = 2n - 2k - 2$

$2n - 5k = 8$ ....(2)

From (1) and (2)

$n = 14$ and $k = 4$

$\therefore n + k = 18$

(73) **(A)** Let the observations be x_1, x_2, x_3, x_4, x_5 and x_6, so

their mean $\bar{x} = \dfrac{\sum_{i=1}^{6} x_i}{6} = 8$

$\Rightarrow \sum_{i=1}^{6} x_i = 8 \times 6 \Rightarrow \sum_{i=1}^{6} x_i = 48$

On multiplying each observation by 3, we get the new observations as $3x_1, 3x_2, 3x_3, 3x_4, 3x_5$ and $3x_6$.

Now, their mean $= \bar{x} = \dfrac{\sum_{i=1}^{6} 3x_i}{6} = \dfrac{3\sum_{i=1}^{6} x_i}{6}$

$\Rightarrow \bar{x} = \dfrac{3 \times 48}{6} = 24$

Variance of new observations

$= \dfrac{\sum_{i=1}^{6} (3x_i - 24)^2}{6} = \dfrac{3^2 \sum_{i=1}^{6} (x_i - 8)^2}{6}$

$= \dfrac{9}{1} \times$ Variance of old observations $= 9 \times 4^2 = 144$

Thus, standard deviation of new observations

$= \sqrt{\text{Variance}} = \sqrt{144} = 12$

(74) **(B)** $p \vee (p \wedge q)$ is equivalent to p.

(75) **(A)** $\int e^{\sin x} \left(\dfrac{x \cos^3 x - \sin x}{\cos^2 x} \right) dx$

$= \int e^{\sin x} \, x \cos x \, dx - \int e^{\sin x} \tan x \sec x \, dx$

$= \int x \, d\left(e^{\sin x}\right) - \int e^{\sin x} \, d(\sec x)$

$= \left\{ x e^{\sin x} - \int e^{\sin x} dx \right\}$

$- \left\{ e^{\sin x} \sec x - \int e^{\sin x} \sec x \cos x \, dx \right\}$

$= x e^{\sin x} - e^{\sin x} \sec x + C$

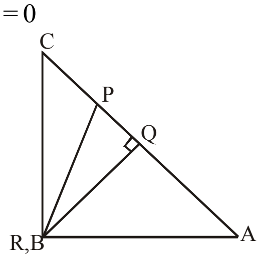
(76) **(D)** $f(x) = x^2 - 4x + a$ always attains its minimum value.

So its range must be closed.

So, $a = \{\phi\}$

(77) **(A)** $2 \sec 4C + \sin^2 2A + \sqrt{\sin B} = 0$

$A = 45°, B = 90°$ and $C = 45°$



Let $AQ = a$, then $BP = \dfrac{a}{2}$,

$PQ = \dfrac{a}{2}$ and $QR = a$

$\therefore PR = \sqrt{a^2 + \dfrac{a^2}{4}} = \dfrac{\sqrt{5}a}{2}$

$\therefore 1 : \alpha : \beta = \dfrac{a}{2} : a : \dfrac{\sqrt{5}a}{2} = 1 : 2 : \sqrt{5}$

$\therefore \alpha = 2$ and $\beta = \sqrt{5}$ $\therefore \alpha^2 + \beta^2 = 9$

(78) **(A)** Starting with 1 | 1 | | | | | 2 3 4 5 6 7 8 9

$= {}^8C_4 = 70$

Starting with 2 | 2 | | | | | 3 4 5 6 7 8 9

$= {}^7C_4 = 35$

Total = 105

$(105)^{th}$ number 26789

(79) **(B)** $(A+B)^2 = (A+B).(A+B) = A^2 + AB + BA + B^2$....... (1)

Since $B = -A^{-1} BA$

$AB = A(-A^{-1}BA) = -(AA^{-1})BA$ [$\because A(BC) = (AB)C$]

$\therefore$ $AB = -BA$ put in (1), we get

$(A+B)^2 = A^2 - BA + BA + B^2$

$(A+B)^2 = A^2 + B^2$

(80) **(B)** $\lim_{x \to 0} \dfrac{(\tan x)^{3/2}[1 - (\cos x)^{3/2}]}{x^{3/2}.x^2}$.

$= 1 \times \lim_{x \to 0} \dfrac{1 - \cos^3 x}{x^2} . \dfrac{1}{1 + (\cos x)^{3/2}}$

$= \dfrac{1}{2} . \dfrac{1}{2}(1 + \cos x + \cos^2 x) = \dfrac{3}{4}$

(81) **(D)** $f_2(x) = f(f(x)) = f(x) = x$

$f_3(x) = f(f_2(x)) = f(x) = x$

$\Rightarrow x^3 - 25x^2 + 175x - 375 = 0$

$(x - 5)(x^2 - 20x + 75)$

$(x - 5)^2 (x - 15) = 0 \Rightarrow x = 5, 15$

(82) **(B)** $n = 3$, P (success) = P (HT or TH) = 1/2

$\Rightarrow p = q = \dfrac{1}{2}$ and $r = 2$

$P(r = 2) = {}^3C_2 \left(\dfrac{1}{2}\right)^2 . \dfrac{1}{2} = \dfrac{3}{8}$

(83) **(B)** $\int_0^{\pi/2} \dfrac{e^{\{\sin x\}} d\{\sin^2 x\}}{\{\sin x\}}$,

where $\{.\}$ denotes the fractional part of x.

$= \int_0^{\pi/2} \dfrac{e^{\sin x} d(\sin^2 x)}{\sin x}$

$0 < x < \pi/2, \Rightarrow 0 < \sin x < 1$ and $\{\sin x\} = \sin x$

$$= \int_0^1 \frac{e^{\sqrt{t}}\,dt}{\sqrt{t}} = 2.$$

(84) **(C)** $L_1 : \frac{x-1}{-2} = \frac{y-0}{1} = \frac{z+1}{1}$;

$L_2 : \frac{x-4}{1} = \frac{y-5}{4} = \frac{z+2}{-1}$

$\vec{V}_1 = -2\hat{i} + \hat{j} + \hat{k},\ \vec{V}_2 = \hat{i} + 4\hat{j} - \hat{k}$

$\cos\theta = \left|\frac{-2+4-1}{\sqrt{6}.\sqrt{18}}\right| = \frac{1}{6\sqrt{3}} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{6\sqrt{3}}\right)$

(85) **(A)** $(\hat{a}.\hat{c})\hat{b} - (\hat{a}.\hat{b})\hat{c} = \frac{1}{\sqrt{2}}\hat{b} + \frac{1}{\sqrt{2}}\hat{c}$

$\therefore\ \hat{a}.\hat{c} = \frac{1}{\sqrt{2}}$ and $\hat{a}.\hat{b} = -\frac{1}{\sqrt{2}}$

$\Rightarrow$ angle between $\hat{a}$ and $\hat{c} = \frac{\pi}{4}$ and angle between $\hat{a}$ and $\hat{b} = \frac{3\pi}{4}$

(86) **(C)** $I = \frac{1}{2}\int_0^{\pi/2} x\,|\cos 2x|\,dx$; $2x = t \Rightarrow dx = \frac{dt}{2}$

$I = \frac{1}{8}\int_0^{\pi} t\,|\cos t|\,dt$

$I = \frac{1}{8}\int_0^{\pi} (\pi - t)\,|\cos t|\,dt$

$2I = \frac{\pi}{8}\int_0^{\pi} |\cos t|\,dt = \frac{2\pi}{8} \Rightarrow I = \frac{\pi}{8}$

(87) **(C)** $x = \tan^{-1} t \Rightarrow \frac{dx}{dt} = \frac{1}{1+t^2}$

$\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = \frac{dy}{dt}(1+t^2)$ .... (1)

$\frac{d^2y}{dx^2} = \frac{d}{dt}\left[\frac{dy}{dt}(1+t^2)\right].\frac{dt}{dx}$

$= \left[\frac{dy}{dt}2t + (1+t^2)\frac{d^2y}{dt^2}\right](1+t^2)$ .... (2)

Hence the given differential equation

$\frac{d^2y}{dx^2} + xy\frac{dy}{dx} + \sec^2 x > 0$, becomes

$(1+t^2)\left[2t\frac{dy}{dt} + (1+t^2)\frac{d^2y}{dt^2}\right]$

$+ y\tan^{-1} t\left[\frac{dy}{dt}(1+t^2)\right] + (1+t^2) = 0$

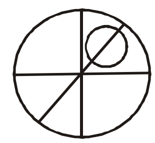
Cancelling $(1 + t^2)$ throughout, we get

$(1+t^2)\frac{d^2y}{dt^2} + (2t + y\tan^{-1} t)\frac{dy}{dt} = -1 \qquad \Rightarrow k = -1$

(88) **(C)** $C_1C_2 = 13$

$r_1 = 30,\ r_2 = 6$

$C_1C_2 < r_1 - r_2$



$\therefore$ The circle $|z_2 - (12+5i)| = 6$ lies with in the circle $|z_1| = 30$

$\therefore$ $\max|z_1 - z_2| = 30 + 13 + 6 = 49$

$\therefore$ Statement-1 is true.

Statement-2 $|z_1 - z_2| \le |z_1| + |z_2|$ is always true.

Equality sign holds if z_1, z_2 origin are collinear and z_1 and z_2 lies on opposite sides of the origin.

$\therefore$ Statement-2 is true.

(89) **(A)** $2\sin^2\theta\, x + \cos^2\theta\, y = 2\cos 2\theta$

Statement-1: The line passes through the point (3, –2)

If $6\sin^2\theta - 2\cos^2\theta = 2\cos 2\theta$

i.e. $6(1 - \cos^2\theta) - 2\cos^2\theta = 4\cos^2\theta - 2$

i.e. $12\cos^2\theta = 8$

$\therefore$ Statement-1 is false.

Statement : $2(1 - \cos^2\theta)x + \cos^2\theta\, y = 4\cos^2\theta - 2$

$\therefore$ $\cos^2\theta(-2x + y - 4) + 2x + 2 = 0$

Family of lines passes through the point of intersection of line $2x - y + 4 = 0$ and $x = -1$

$\therefore$ The point is (–1, 2)

$\therefore$ Statement-2 is true.

(90) **(A)** $f(x) = \frac{1}{1-\sin x}$ and $f(-x) = \frac{1}{1+\sin x}$

$\therefore\ I = \int_{-\pi/4}^{\pi/4} \frac{dx}{1+\sin x}$

Now, $f(x) + f(-x) = 2I = \int_{-\pi/4}^{\pi/4} \frac{2\,dx}{1-\sin^2 x}$

$\Rightarrow I = \int_{-\pi/4}^{\pi/4} \frac{dx}{\cos^2 x}$. This is an even function.

$\therefore\ I = 2\int_0^{\pi/4} \sec^2 x\,dx \ne 0 \Rightarrow$ Statement-1 is false.